# MORE EXERCISES ON NATURAL DEDUCTION AND TRUTH TABLES

SINA HAZRATPOUR

In the following problems, $P, Q, R, \ldots$ are propositional variables.

**Exercise 1.** Give a natural deduction proof of the following propositional formulas:

(1) $P \vee Q \Leftrightarrow Q \vee P$
(2) $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
(3) $P \Rightarrow \neg\neg P$
(4) $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$
(5) $(P \wedge Q) \Rightarrow ((P \Rightarrow R) \Rightarrow \neg(Q \Rightarrow \neg R))$
(6) $(P \vee (Q \wedge P)) \Rightarrow P$

**Exercise 2.** Two propositional formulas, $A$ and $B$, are said to be *logically equivalent* if $A \Leftrightarrow B$ is provable. Show that the following propositional formulas are logically equivalent.

(1) $P \vee Q$ and $Q \vee P$
(2) $(P \vee Q) \vee R$ and $P \vee (Q \vee R)$

With the help of the law of double negation, prove that the following propositional formulas are logically equivalent. Remember you need to prove these in natural deduction style.

(1) $\neg P \wedge \neg Q$ and $\neg(P \vee Q)$
(2) $\neg P \vee \neg Q$ and $\neg(P \wedge Q)$

Can you do any of the deductions without the help of the law of double negation?

The the last two logical equivalency is known as *De Morgan's Laws*. They were first formulated by the British mathematician and logician Augustus De Morgan in the 19th century.

**Exercise 3.** Use a truth table for each of the following propositional formulas to verify the fact that they are tautologies.

(1) $(((P \Rightarrow Q) \Rightarrow Q) \Rightarrow Q) \Rightarrow P \Rightarrow Q$
(2) $(P \Rightarrow (Q \wedge R)) \Rightarrow (P \Rightarrow Q) \wedge (P \Rightarrow R)$
(3) $\neg(((\neg P \Leftrightarrow P) \Leftrightarrow P) \Leftrightarrow P)$

**Exercise 4.** Use appropriate truth tables to show that the following propositional formulas are not tautologies by observing that not all the rows of the truth tables consist solely of 1's (We used 1 to denote the truth value "`true`").

- $(P \Rightarrow Q) \Rightarrow Q \Rightarrow P$
- $(P \Rightarrow Q) \Rightarrow \neg P \Rightarrow \neg Q$