MATH 301: INTRODUCTION TO PROOFS
HOMEWORK 1

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Problem 1. Prove that, for all real numbers x and y, if x is irrational, then $x+y$ and $x-y$ are not both rational.

Problem 2. Suppose a and n are non-zero natural number. Show that the proposition

"If $a^n - 1$ is prime, then n is prime"

is not necessarily true. What other condition about a can we add to the conclusion so that the proposition becomes valid? Then give a proof of the modified proposition.

Problem 3. Let P, Q and R be propositions.

(1) Construct a proof of proposition

$$P \vee (Q \wedge R) \Rightarrow (P \vee Q) \wedge (P \vee R)$$

using natural deduction.

(2) Draw the associated tree of your deduction in the first part.

Problem 4. In this exercise we'll learn about Peirce's law, a curiosity of *classical* logic. Let P, Q and R be propositions.

(1) With the help of the axiom of double negation (which says for any proposition A, the proposition $A \Leftrightarrow \neg\neg A$ is a tautology) construct a proof of proposition

$$((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$$

in the style of natural deduction.

(2) Verify the result of part 1 by drawing a truth table.

Problem 5. On an island inhabited by knights and knaves, where the former always tell the truth and the latter always lie, you meet three individuals: Alice, Bob, and Eve. Alice says that Bob is a knight. Bob say that Alice is a knight but Eve is a knave. Eve says that both Alice and Bob are knights. Determine who is a knight and who is a knave by constructing a truth table.

Problem* 6. Show that there are irrational numbers x and y such that x^y is a rational number.