MATH 301: INTRODUCTION TO PROOFS
HOMEWORK 2

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Problems.
  §1.E (Chapter 1 Exercises) ‖ 1.8, 1.10, 1.18

Problem 1. Find logical formulae that represent each of the following statements.
  (1) If someone is alone, they are not with someone else.
  (2) No mortal man can slay every dragon.
  (3) There is no greatest odd integer.
  (4) There is an integer that is divisible by every integer.
  (5) If an integer has a rational square root, then that root is an integer.
  (6) There is no smallest positive real number.

Problem 2. Find the statements in plain English, involving as few variables as possible, that are represented by each of the following logical formulae.
  (1) $\forall d \in \mathbb{N}, [(\exists q \in \mathbb{Z}, n = qd) \Rightarrow (d = 1 \vee d = n)]$.
  (2) $\forall a \in \mathbb{R}, [a > 0 \Rightarrow (\exists b \in \mathbb{R}, (b > 0 \wedge a < b))]$.
  (3) $\exists a \in \mathbb{Z}, [(\exists q \in \mathbb{Z}, 2q = a) \wedge (\exists q \in \mathbb{Z}, 2q + 1 = a)]$.

Problem 3. Prove the assertions (3)-(6) of Problem 1 above using the structure of the logical formulae you obtained. Can you identify the proof techniques and rules of inference that you used in your proofs?

Problem 4. Give a natural deduction proof of
$$\forall x \ (A(x) \Rightarrow B(x)) \Rightarrow (\forall x \ A(x) \Rightarrow \forall x \ B(x)).$$