

**MATH 301: INTRODUCTION TO PROOFS  
HOMEWORK 3**

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**Problems.**

§2.E (Chapter 2 Exercises) || 2.6, 2.14, 2.15, 2.16

**Problem 1.** Let  $\{A_{i,j} \mid i \in I, j \in J\}$  be a family of sets. Prove that

$$(1) \bigcup_{i \in I} \bigcup_{j \in J} A_{i,j} = \bigcup_{j \in J} \bigcup_{i \in I} A_{i,j}$$

$$(2) \bigcap_{i \in I} \bigcap_{j \in J} A_{i,j} = \bigcap_{j \in J} \bigcap_{i \in I} A_{i,j}$$

**Problem 2.** Let  $X, Y$  be classical sets. Prove that  $X \setminus (X \setminus Y) = X \cap Y$ . Can we drop the condition of being *classical* about either  $X$  or  $Y$  and have the same conclusion? If so, which one?

**Problem 3.** Prove the De Morgan's laws for classical sets  $A, X, Y$  and a family of classical sets  $\{X_i \mid i \in I\}$ .

$$(1) A \setminus (X \cup Y) = (A \setminus X) \cap (A \setminus Y)$$

$$(2) A \setminus (X \cap Y) = (A \setminus X) \cup (A \setminus Y)$$

$$(3) A \setminus \bigcup_{i \in I} X_i = \bigcap_{i \in I} (A \setminus X_i)$$

$$(4) A \setminus \bigcap_{i \in I} X_i = \bigcup_{i \in I} (A \setminus X_i)$$